Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series
Integral Calculus #7

Properties of Definite Integrals

Recall that the definite integral:

$$\int_{x=a}^{x=b} f(x)\,dx$$

is the total change between $x = a$ and $x = b$, in the quantity with rate of change $f(x)$.

So if $F' = f$ then

$$\int_{x=a}^{x=b} f(x)\,dx = F(b) - F(a)$$

We described this as the total amount of water flowing down a pipe from time a to time b when the flow meter reading is $f(x)$.

We can simplify the notation to:

$$\int_{a}^{b} f(x)\,dx = F(b) - F(a)$$

Using the water-in-the pipe analogy makes it easy to see that the following properties are true.

Two pipes – same interval:

$$\int_{a}^{b} f(x) + g(x)\,dx = \int_{a}^{b} f(x)\,dx + \int_{a}^{b} g(x)\,dx$$

One pipe – two intervals:

$$\int_{a}^{b} f(x)\,dx + \int_{b}^{c} f(x)\,dx = \int_{a}^{c} f(x)\,dx$$